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ABSTRACT

Rotation is used in almost all exploratory factor analysis (EFA) studies. There are numerous rotation strategies that can be employed in these various applications. This paper reviews the various rotation choices in EFA studies, including confirmatory rotation, and presents criteria useful in selecting rotation methods in various analytic situations. Factors are rotated in attempts to improve the interpretability of results. The first decision the researcher must make is whether he or she wants the factors to be correlated (oblique rotation) or uncorrelated (orthogonal rotation). As a general rule, if the researcher is primarily interested in getting results that best fit the data, the factors should be rotated obliquely. If the interest is in the generalizability of results, then orthogonal rotation should be conducted instead. It is suggested that regardless of the magnitude of the correlation among the factors it is unnecessary to do oblique rotation in addition to orthogonal rotation. The correlation among the factors yields results that are more difficult than results of an orthogonal rotation, and the slight difference between results of an oblique and an orthogonal rotation is nearly insignificant. (Contains 7 tables and 10 references.) (SLD)

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Exploratory and Confirmatory Rotation Strategies in Exploratory Factor Analysis

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Abstract

Rotation is used in almost all exploratory factor analysis studies. There are numerous rotation strategies that can be employed in these various applications. The present paper reviews the various rotation choices in EFA studies, including confirmatory rotation, and presents criteria useful in selecting rotation methods in various analytic situations.

Factor analysis is a variable reduction technique which allows us to simplify our data by combining numerous variables into a much smaller set of synthetic variables called factors. Factor analysis is "designed to identify factors, or dimensions, that underlie the relations among a set of observed variables" (Pedhazur & Schmelkin, 1991, p. 66). As Tinsley and Tinsley (1987) noted:

> Factor analysis is an analytic technique that permits the reduction of a large number of interrelated variables to a smaller number of latent or hidden dimensions. The goal of factor analysis is to achieve parsimony by using the smallest number of explanatory concepts to explain the maximum amount of common variance in a correlation matrix. (p. 414)

The ability of factor analysis to detect underlying constructs makes it an extremely useful tool for researchers who want to demonstrate that their results have construct validity. Pedhazur and Schmelkin (1991) noted that correlations among indicators (i.e., variables):

> . . . are attributed to the construct they are said to reflect. Therefore, to accept the validity of the model, hence the validity of the indicators of the construct, it is necessary, although not sufficient, to demonstrate that it is consistent

> with the data. . . At a minimum, it is necessary to demonstrate that the reflective indicators of a construct (e.g., items, subtests) "hang together;" that they are homogeneous. . . Of various approaches for studying the internal structure of a set of indicators, probably the most useful is some variant of factor analysis. . . (p. 66)

Similarly, Gorsuch (1983) stated that, "a prime use of factor analysis has been in the development of both the operational constructs for an area and the operational representatives for the theoretical constructs" (p. 350). Hetzel (in press) also stated that, "factor analysis has perhaps been most useful in testing theories about structure, and evaluating the construct validity of scores used in substantive inquiry."

It is important to note that researchers can use factor analysis in both theory development (i.e., exploratory factor analysis) and theory evaluation (i.e., confirmatory factor analysis). The scenario below provides an example of when factor analysis can be used in theory development:

> Suppose, for the sake of illustration, that a researcher is interested in constructing a measure of self-concept and that he or she writes (or selects from the literature) 20 items on which respondents are asked to rate themselves. Assume further that the researcher wishes to add the

> ratings on the 20 items in order to arrive at a total self-concept score. It stands to reason that, for a total score to be meaningful, it is essential for the items to "hang together;" to tap the same dimension. (Pedhazur & Schmelkin, 1991, p. 67)

Furthermore, "exploratory factor analysis is *not*, or should not be, a blind process in which all manner of variables or items are thrown into a factor analytic 'grinder' in the expectation that something meaningful will emerge" (Pedhazur & Schmelkin, 1991, p. 591). "Exploratory factor analysis is concerned with the question of how many factors are necessary to explain the relations among a set of indicators and with the estimation of the factor loadings" (Pedhazur & Schmelkin, 1991, p. 67). Researchers interested in testing hypotheses about the underlying construct will instead opt to do confirmatory factor analysis (CFA).

The basic mathematical computations behind both EFA and CFA are related (Thompson, 1997). Exploratory factor analysis and CFA only differ in that the researcher doing an EFA "seldom selects the variables so that the principal factors--or any other factors given by the initial solutions are of theoretical interest" (Gorsuch, 1983, p. 176). Confirmatory factor analysis "is concerned with parameter estimation and tests of hypotheses regarding, for example, the number of factors underlying the

relations among a set of indicators" (Pedhazur & Schmelkin, 1991, p. 67). For instance:

> Suppose, then, it is desired to carry out a confirmatory factor analysis of indicators (e.g., items) of Self-Concept. Unlike exploratory factor analysis, in confirmatory factor analysis it is, obviously necessary to advance hypotheses to be tested. For illustrative purposes, it will be assumed that Self-Concept is hypothesized to be comprised of two correlated dimensions--Academic and Social. . . (Pedhazur & Schmelkin, 1991, p. 70)

Why Rotate the Factors

While the results of a factor analysis may produce a:

> good fitting solution [the result] is not necessarily susceptible to a meaningful interpretation. It is in attempts to improve the interpretability of results that factors are rotated. . . Because there is an infinite number of ways in which factors can be transformed or rotated, the question arises: Are some rotations better than others? Criteria for determining what are better rotations are not always clear or agreed upon. The reason is that rotations are resorted to for the purpose of improving

> interpretability of factor-analytic results, and interpretability is, by its very nature, inextricably intertwined with theory. What might be viewed as a meaningful rotation from one theoretical perspective may not be considered meaningful, even utterly inappropriate, from another. (Pedhazur & Schmelkin, 1991, p. 611)

Thus, the present paper will review the various rotation choices in EFA studies, including confirmatory rotation EFA studies (Thompson, 1992), and will present criteria with which to select rotation methods in various analytic situations. In addition, data from the Holzinger and Swineford (1939) study will be used to illustrate the various rotation procedures. Table 1 lists the variable names, means, and standard deviations from the nine variables used in the examples throughout the present paper.

Guidelines for Rotation

For the factors to be rotated, "a principle for guiding the rotation is needed" (Gorsuch, 1983, p. 176). Although many different sets of criteria have been suggested, Thurstone's simple structure "has had the greatest impact on the development of various rotational approaches aimed at improving interpretability" (Pedhazur & Schmelkin, 1991, p. 612). Thurstone's (1947, p. 335) guidelines for simple structure--rotating to the most parsimonious position are as follows:

1. Each variable should have at least one zero factor coefficient.

2. Each factor should have a set of variables whose factor coefficients are zero.

3. For every pair of factors, there should be several variables whose factor coefficients are zero for one factor but not for the other.

4. For every pair of factors, a large proportion of the variables should have zero factor coefficients on both factors whenever more than about four factors are extracted.

5. For every pair of factors, there should only be a small number of variables with nonzero factor coefficients on both.

Thurstone's goal in developing his set of guidelines for rotating factors was that "the factor pattern of any given variable would be constant when the variable was included in another factor analysis containing the same common factors" (Gorsuch, 1983, p. 177). This leads to findings that are more replicable across studies. As Gorsuch (1983) noted, "Thurstone showed that such rotation leads to a position being identified for each factor that would be independent of the number of variables defining it. Therefore, a simple structure factor should be relatively invariant across studies" (p. 177).

Oblique versus Orthogonal Rotation

The first decision that a researcher must make is to decide whether he/she wants the factors to be correlated (i.e., oblique rotation) or uncorrelated with one another (i.e., orthogonal rotation). Tables 2 and 3 show factor correlation matrices for correlated and uncorrelated factors respectively. There are advantages and disadvantages to using either rotation procedure and the researcher's decision depends upon the researcher's goals. As a general rule, if a researcher is primarily concerned with getting results that "best fit" his/her data, then the researcher should rotate the factors obliquely. If on the other hand the researcher is more interested in the generalizability of his/her results, then orthogonal rotation should be conducted instead. However, results from an oblique rotation and an orthogonal rotation almost always produce generally similar results. Therefore, orthogonal rotation is almost always the preferred choice. Unless the results from an oblique rotation drastically differ (i.e., radically improve interpretation) from those of an orthogonal rotation, oblique rotation should generally not be done. The purpose of research is to enhance and promote knowledge in the area being investigated, therefore, being able to generalize one's findings becomes of utmost importance to researchers.

However, it should be noted that there are those who do not believe that orthogonal rotation is necessarily better than

oblique rotation. Cattell (1978) stated that researchers who choose to do orthogonal rotation do so because ". . . in half of these cases it is evidently done in ignorance of the issue rather than by deliberate intent" (p. 128). Similarly, Thurstone (1947) noted that researchers' use of orthogonal rotation represents "our ignorance of the nature of the underlying structure of mental traits. . . The reason for using uncorrelated reference traits can be understood, but it cannot be justified" (p. 139).

Both Cattell (1978) and Thurstone (1947) have valid arguments. Unfortunately, they have both made the same inaccurate assumption that researchers who choose orthogonal rotation do so because of ignorance. This is simply not true. Orthogonal rotations are recommended because "There is no denying that orthogonal rotations have the advantage of simplicity" (Pedhazur & Schmelkin, 1991, p. 615) and because the results of orthogonal rotation are more replicable. As stated before, the purpose of rotation is to obtain results that are parsimonious and more likely to be replicated by future researchers. It was never suggested that orthogonal rotation provides results that are more representative of the nature or reality. It will be shown later on that this is one drawback of orthogonal rotation. However, it is believed that the benefits far outweigh the costs of orthogonal rotation. Additionally, Hetzel (in press) stated that, ". . . when simple structure is clear, standard rotation procedures can be expected to produce similar interpretations."

There are two elements which effect whether oblique and orthogonal rotation will produce similar results: (a) the factor to variable ratio and (b) the degree of correlation between the factors. When the factor to variable ratio is small, both oblique and orthogonal rotation will produce similar results. When this ratio is small, simple structure will tend to be the same regardless of which type of rotation is used. In addition, the smaller the correlation (i.e., the closer to zero) between the factors, the more likely oblique and orthogonal rotation will produce similar results.

After deciding upon either oblique or orthogonal rotation, researchers must make a second decision. Within both oblique and orthogonal rotation, there are many different rotation techniques that may be used. For example, if a researcher does an oblique rotation, the researcher can choose to do an oblimin or a promax rotation procedure. Within orthogonal rotation, a researcher may choose to do a varimax or quartimax or some other orthogonal procedure. There are many types of orthogonal and oblique rotational strategies (e.g., maxplane, orthomax, or equamax) from which a researcher can choose. The procedures listed above are the most widely used.

Oblique Rotation

As previously mentioned, there are disadvantages and advantages to using either procedure. An oblique rotation is useful for two reasons. To begin with, an oblique rotation is

more representative of the nature of reality. This is because an oblique rotation allows the factors to be correlated with one another. In the real world it is very unlikely that the factors would have a zero correlation with one another. Likewise, Pedhazur and Schmelkin (1991) stated, ". . . we believe that such solutions (orthogonal) are, in most instances naive, unrealistic, portrayals of behavioral science phenomena" (p. 615). For example, if after conducting an factor analysis on a set of questions, a researcher came up with a quantitative factor and a verbal factor, it is very unlikely that the correlation coefficient for the two factors will be zero. Instead, it is expected that these two factors will probably be somewhat correlated with each other. Unlike orthogonal rotation, an oblique rotation allows this correlation to be incorporated into the analysis.

A second advantage (or disadvantage depending on the researcher's goal) of an oblique rotation is that oblique rotation provides results that "best fit your data". In other words, oblique rotation is strongly influenced by sample specific measurements (e.g., degree of correlation between the factors). The degree of correlation between factors will vary from one study to the next--no two studies will have identical factor matrices.

As a consequence, the results obtained by an oblique rotation will be less likely to be replicated by future studies.

This occurs because of sampling error. In an oblique rotation, the researcher is estimating a greater number of coefficients than in an orthogonal rotation, so there are more opportunities for sampling error to influence the results. The factor pattern matrix is not equal to the factor structure matrix in an oblique rotation. Hetzel (in press) described the elements in factor pattern matrix as "weights (equivalent to beta-weights in multiple regression analysis) that can be applied to the 'observed' variables based on perceptions of how the constructs delineate the 'observed' scores." The factor structure matrix is comprised of the bivariate correlations between the variables and the factors. Since the factors are correlated with one another, the factor pattern matrix and the factor structure matrix will not be the same. However, the factor structure matrix and the factor pattern matrix are identical in an orthogonal rotation in which case the researcher is estimating fewer coefficients than in an oblique rotation.

Another problem with the results of an oblique rotation is that the interpretation is more complex. The researcher is not only required to provide an explanation of the underlying factor structure but must also explain the correlations among the factors (Hetzel, in press).

Oblimin

Oblimin is a classic example of orthogonal rotation. The goal of an oblimin rotation is to obtain simple structure while

allowing the factors to be correlated with each other. The magnitude of the correlation between the factors can be set by the researcher. This suggests that some thought and effort is required on the part of the researcher. The researcher should have a theoretically-based hypothesis concerning the degree of correlation between the factors otherwise the results may not honor reality. Tables 4 and 5 show the factor structure and factor pattern matrices from an oblimin rotation.

Promax

Promax can be considered to be a hybrid of oblique and orthogonal rotation. It is an alteration of an orthogonal rotation that yields an oblique solution. There are three steps to promax rotation. The first step requires that the sample matrix be rotated using an orthogonal rotation (e.g., varimax or quartimax). In the second step, a target matrix is obtained. The target matrix is computed by raising the coefficients to an exponent (e.g., 3 or 4). When this is done, the obtained coefficients will become smaller. The smaller the original coefficient, the smaller the obtained coefficient will get. For example, if the original coefficient was .1 and was cubed, the obtained coefficient will be .001. However, if the original coefficient was .9 and was then cubed, the obtained coefficient will be .73. The gap between the larger coefficients and the smaller coefficients will increase as a result of this mathematical manipulation. Please note that when using an even

number the sign of the obtained coefficient must be changed. Third, a "Procrustes" rotation is done, i.e., the orthogonally rotated matrix from step one is rotated to the "best fit" position with the target matrix in step two.

If a researcher chooses to do an oblique rotation, promax seems to be the optimal choice. Although an oblique rotation by its nature will provide in results that are not likely to be replicated by future studies, promax appears to offer a solution that will be more replicable than an oblimin rotation. The target matrix that is generated in the second step of a promax rotation is representative of matrices from other studies. Thus, when the sample matrix is rotated to "best fit" the target matrix, this may make the results of promax more replicable.

Orthogonal Rotation

By now some of the advantages and disadvantages of using orthogonal rotation should be obvious. While oblique rotation provides a result that "best fits" the researcher's sample data, orthogonal rotation provides a solution that "best fits" past and future data. In essence, the results of an orthogonal rotation are more likely to be replicated in future studies and to have been found by previous investigators. This occurs because there is less sampling error in an orthogonal rotation, since an orthogonal rotation also produces results that are more parsimonious.

Additionally, since the factors are uncorrelated with each other, the interpretation of orthogonally rotated factors is much simpler than that of obliquely rotated factors. Hetzel (in press) noted:

> with an oblique rotation it is important to consult the factor structure coefficients in addition to the factor pattern coefficients. This is unnecessary after orthogonal rotation, since the orthogonality of the factors meant that the factor pattern coefficients and structure coefficients were identical.

Unfortunately, while the results may be easier to explain, the results do not honor the nature of reality.

Quartimax

Quartimax tends to "clean up the variables" (Stevens, 1996, p. 368). The factor pattern of a variable is simplified by forcing a given variable to correlate highly on one main factor and either not at all or very low on other factors. Quartimax rotation tends to result in one general factor because each variable tends to correlate primarily with one factor (Stevens, 1996). The variable is much easier to interpret in this type of rotation. Table 6 shows the results from a quartimax rotation.

Varimax

Varimax focuses on cleaning up the factors. Varimax rotation produces factors that have high correlations with one smaller set

of variables and little or no correlation with another set of variables (Stevens, 1996). Thus, interpretation of the factors is easier than in a quartimax rotation. With varimax rotation there is a tendency for the principal factor to disappear because the factor variance is redistributed (Hetzel, in press). Table 7 provides an example of varimax rotation.

Since the purpose of rotation is to simplify interpretation of the factors, it may appear, at first, that the logical choice is varimax. However, other elements need to be taken into consideration. For instance, if the researcher believes that there will be one general factor that accounts for most of the variance, then quartimax becomes the logical choice. If there is no reason for the researcher to suspect a principal factor, then varimax is the most appropriate choice. Please note, however, that the results from quartimax and varimax will tend to similar (see Tables 6 & 7).

Conclusion

Pedhazur and Schmelkin (1991) noted that:

> From the perspective of construct validation, the decision whether to rotate factors orthogonally or obliquely reflects one's conception regarding the structure of the construct under consideration. It boils down to the question: Are aspects of a postulated multidimensional construct intercorrelated? The answer to this question is

> relegated to the status of an assumption when an orthogonal rotation is employed. . . The preferred action is, in our opinion, to rotate both orthogonally and obliquely. When on the basis of the latter, it is concluded that the correlations among the factors are negligible, the interpretation of the simpler orthogonal becomes tenable. (p. 615)

However, it is suggested that regardless of the magnitude of the correlation among the factors that it is unnecessary to do oblique rotation in addition to orthogonal rotation. The correlation among the factors yields results that are more difficult to interpret than results of an orthogonal rotation. Moreover, the slight difference between results of an oblique rotation and an orthogonal is virtually insignificant. Thus, it becomes much easier and simpler to rotate the factors orthogonally.

References

Cattell, R. B. (1978). The scientific use of factor analysis in behavioral and life sciences New York: Plenum.

Gorsuch, R. L. (1983). Factor analysis (2nd ed.). Hillsdale, NJ: Erlbaum.

Hetzel, R. D. (in press). A primer on factor analysis with comments on patterns of practice and reporting. In B. Thompson (Ed.), Advances in social science methodology (Vol. 4). Greenwich, CT: JAI Press.

Holzinger, K. J., & Swineford, F. (1939). A study in factor analysis: The stability of a bi-factor solution (No. 48). Chicago, IL: University of Chicago.

Pedhazur, E. J., & Schmelkin, L. P. (1991). Measurement, design, and analysis: An integrated approach Hillsdale, NJ: Erlbaum.

Stevens, J. (1996). Applied multivariate statistics for the behavioral sciences (3rd ed.). Mahwah, NJ: Erlbaum.

Thompson, B. (1992). A partial test distribution for cosines among factors across samples. In B. Thompson (Ed.), Advances in social science methodology (Vol. 2, pp. 81-97). Greenwich, CT: JAI Press.

Thompson, B. (1997). The importance of structure coefficients in structural equation modeling confirmatory factor analysis. Educational and Psychological Measurement, 57, 5-19.

Thurstone, L. L. (1947). Multiple-factor analysis Chicago: Chicago University Press.

Tinsley, H. E. A., & Tinsley, D. J. (1987). Uses of factor analysis in counseling psychology research. Journal of Counseling Psychology, 34, 414-424.

Table 1
Variable Names and Descriptive Statistics

Variable	Mean	Std Dev	Label
T6	9.18272	3.49235	PARAGRAPH COMPREHENSION TEST
T7	17.36213	5.16189	SENTENCE COMPLETION TEST
T9	15.29900	7.66922	WORD MEANING TEST
T10	96.27575	25.05927	SPEEDED ADDITION TEST
T12	110.54153	20.25230	SPEEDED COUNTING OF DOTS IN SHAPE
T13	193.46844	36.32946	SPEEDED DISCRIM STRAIGHT AND CURVED CAPS
T14	175.15282	11.50753	MEMORY OF TARGET WORDS
T15	90.00997	7.72937	MEMORY OF TARGET NUMBERS
T17	8.23256	4.91587	MEMORY OF OBJECT-NUMBER ASSOCIATION TARG

Table 2
Correlated Factor Correlation Matrix

Factor	Factor I	Factor II	Factor III
I	1.00000		
II	.19734	1.00000	
III	.13433	.19072	1.00000

Table 3
Uncorrelated Factor Correlation Matrix

Factor	Factor I	Factor II	Factor III
I	1.00000		
II	.00000	1.00000	
III	.00000	.00000	1.00000

Table 4
Oblimin Factor Structure Matrix

Variable	Factor I	Factor II	Factor III
T6	.89709	.18874	.18235
T7	.90907	.18352	.06319
T9	.89075	.19582	.13819
T10	.11973	.78262	.21840
T12	.12790	.83340	.08979
T13	.26793	.71416	.13282
T14	.23261	.06934	.79169
T15	.00766	.08406	.78183
T17	.12407	.42109	.67476

Table 5
Oblimin Factor Pattern Matrix

Variable	Factor I	Factor II	Factor III
T6	.88837	.00146	.06274
T7	.91452	.01496	-.06251
T9	.88504	.01814	.01584
T10	-.04378	.77674	.07614
T12	-.03115	.85264	-.06864
T13	.13384	.69098	-.01694
T14	.14801	-.11111	.79300
T15	-.09015	-.05144	.80375
T17	-.01963	.30694	.61885

Table 6
Quartimax Factor Pattern/Structure Matrix

Variable	Factor I	Factor II	Factor III
T6	.88654	.09644	.11579
T7	.90563	.09979	-.00583
T9	.88175	.10792	.07045
T10	.03439	.77380	.13901
T12	.04465	.83573	.00249
T13	.19670	.69706	.04998
T14	.18814	-.01534	.78670
T15	-.04210	.02116	.78814
T17	.04957	.36501	.63929

Table 7
Varimax Factor Pattern/Structure Matrix

Variable	Factor I	Factor II	Factor III
T6	.88632	.09442	.11913
T7	.90577	.09867	-.00241
T9	.88167	.10625	.07386
T10	.03496	.77269	.14494
T12	.04570	.83563	.00892
T13	.19744	.69641	.05581
T14	.18580	-.02151	.78712
T15	-.04440	.01528	.78815
T17	.04814	.36013	.64217